SYJC – MARCH 2016
MATHEMATICS

(1hr 30 minute)
(80 Marks)
Date: 26.12. 2015

Ans. 1. (i) p → q

= ~ $p \vee q$

Dual : ~ $p \vee q$

Negation : ~ $p \vee q$

(ii) Given f is Continuous at x = 2

$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

$\therefore \lim_{x\to 2^-} \frac{x^2+5}{x-1} = \lim_{x\to 2^+} (kx+1)$

$\therefore \frac{4+5}{2-1} = k(2)+1$

$\therefore 9 = 2k+1$

$\therefore k = 4$

(iii) (5x-4) is polynomial function,hence is continuous for all x€ R

$x^2 - 4$ is polynomial function,hence is continuous for all x € R

Hence,f(x) is a rational function and is continuous in the domain except where denominator = 0

i.e when $x^2 - 4 = 0$

i.e $x^2 = 4$ i.e x = $\underline{+}2$

∴ f(x) is continuous for all real values except at x = -2 and x = 2

(iv) As given

$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\therefore \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\therefore \begin{bmatrix} 2x+3 & 10-4 \\ 14+1 & 2y-6+2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\therefore \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Using equality of two matrices, we get

$2x + 3 = 7$

and $2y - 4 = 14$

$\therefore 2x = 4$

$\therefore x = 2$

and $2y = 18$

$\therefore y = 9$

$\therefore x = 2$ *and* $y = 9$ *are the required values.*

(v) Given line y=x

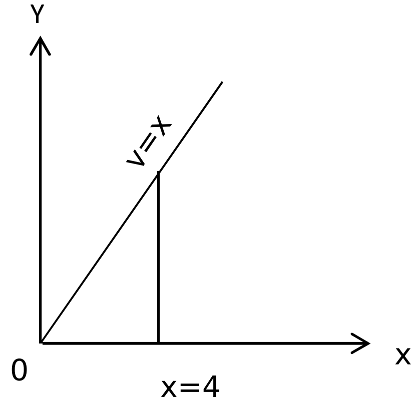
Required volume is

V = $\pi \int_a^b y^2\, dx$

= $\pi \int_0^4 x^{2\,dx}$ = $\frac{\pi}{3}[x^3]_0^4$

V = $\frac{\pi}{3}[4^3 - 0]$ = $\frac{\pi}{3}$ (64)

∴ Required volume is $\frac{64\pi}{3}$ cubic units.



(vi) Given $x = \frac{2t}{1+t^2}$ and $y = \frac{1-t^2}{1+t^2}$

Here t us the pafameter.

Put t = $\tan\theta$, in both the expressions.

∴ $x = \sin 2\theta$

∴ $\frac{dy}{d\theta}$ = 2 $\cos 2\theta$ .....(1)

and $y = \cos 2\theta$

∴ $\frac{dy}{d\theta}$ = -2 $\sin 2\theta$

∴ $\frac{dy}{dx}$ $\frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$, $\frac{dy}{d\theta} \neq 0$

= $\frac{2\ \sin 2\theta}{2\ \cos 2\theta}$

= - $\frac{x}{y}$

(vii) y = $tan^{-1}\left[\frac{4x}{1+5x^2}\right]$

= $tan^{-1}\left[\frac{5x-x}{1+(5x)(x)}\right]$

= tan^{-1} (5x) - $tan^{-1}x$ $\left[\because tan^{-1}a - tan^{-1}\left(\frac{a-b}{1+ab}\right)\right]$

Differentiating w.r.t.x

∴ $\frac{dy}{dx} = \frac{1}{1+x^2}$ (5) - $\frac{1}{1+x^2}$ = $\frac{5}{1+25x^2}$ - $\frac{1}{1+x^2}$

(viii) Given C = $x^3 - 16x^2$ + 47x

∴ Average cost C_A = $\frac{C}{x}$

C_A = $x^2 - 16x + 47$

Differentiating w.r.t.x

$$\frac{dC_A}{dx} = 2x - 16$$

Now C_A $is\ decreasing\ if$ $\frac{dC_A}{dx} < 0$

that is $2x - 16 < 0$

∴ $x < 0$

Average cost is decreasing for $x < 8$

Ans: 2. (A) (i) Converse : If the farmers are happy then the monsoon is good.

Contrapositive : If farmers are not happy then the monsoon is not good.

Inverse : If monsoon is not good then farmers are not happy.

(ii)

p	q	q → p	P → (q →p)
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

$\sim P$	$\sim P$ v q	$\sim p \rightarrow (\sim p \vee q)$
F	T	T
F	F	T
T	T	T
T	T	T

From the above truthtable,since the truth values are identical,the statements are equivalent.

(iii) $\int \frac{2x-1}{(x-1)(x+2)(x-3)} \ dx$

Now $\frac{2x-1}{(x-1)(x+2)(x-3)}$

= $\frac{A}{(x-1)}$ + $\frac{A}{(x+2)}$ + $\frac{c}{(x-3)}$ .. (∝)

2x -1 = A $(x+2)(x-3) + B(x-1)(x-3)$

+ C $(x-1)(x+2)$ ….(i)

Putting x = 1 in equation (i), we get A = $\frac{-1}{6}$

Putting x = -2 in equation (i), we get B = $\frac{1}{3}$

Putting x = 3 in equation (i), we get c = $\frac{1}{2}$

Substituting the values of A, B, and C in equation (∝),

we get

$$\frac{2x-1}{(x-1)(x+2)(x-3)} = \frac{\frac{-1}{6}}{(x-1)} + \frac{-\frac{1}{3}}{(x-2)} + \frac{\frac{1}{2}}{(x-3)}$$

$$\int \frac{2x-1}{(x-1)(x+2)(x-3)}$$

$$= \frac{-1}{6}\int \frac{dx}{(x-1)} - \frac{1}{3}\int \frac{dx}{(x-2)} + \frac{1}{2}\int \frac{dx}{(x-3)}$$

$$= \frac{-1}{6}\, log|x-1| \; - \frac{1}{3}\, log|x+2| + \frac{1}{2}\, |x-3| + c$$

(B) (i) $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{e^{8x} - e^{5x} - e^{3x} + 1}{\cos 4x - \cos 10x}$,

$$= \lim_{x\to 0} \frac{e^{5x} - (e^{3x}-1) - (e^{3x}-1)}{-2\sin\left(\frac{4x+10x}{2}\right).\sin\left(\frac{4x-10x}{2}\right)}$$

$$= \lim_{x\to 0} \frac{(e^{5x}-1)(e^{3x}-1)}{+2\sin 7x\,.\sin 3x}$$

Dividing numerator and denominator by x^2

$$= \lim_{x\to 0} \frac{\frac{e^{5x}-1}{5x}5.\frac{e^{3x}-1}{3x}.3}{2\frac{\sin 7x}{x}.\frac{\sin 3x}{x}}$$

$$= \frac{1}{2} \lim_{x\to 0} \frac{\frac{e^{5x}-1}{5x}5.\frac{e^{3x}-1}{3x}.3}{2\frac{\sin 7x}{x}.7\frac{\sin 3x}{x}.3}$$

$$= \frac{1}{2}\, \frac{5\log e.3\log e}{7(1).3\,(1)}$$

$$= \frac{1}{2} \times \frac{15}{21} = \frac{15}{42} = \frac{5}{14}$$

$$\therefore f(0) = \frac{5}{14}$$

$$\therefore f(0) \neq \lim_{x\to 0} f(x)$$

Hence $f(x)$ is discontinuous at $x = 0$

(ii) $y = \sqrt{\frac{(3x-4)^3}{(x+1)^4(x+2)}}$

taking logarithm of both the sides, we get

$$\log y = \frac{1}{2}\left[\log \frac{(3x-4)^3}{(x+1)^4(x+2)}\right]$$

$$\log y = \frac{1}{2}\ [3\log(3x-4) - 4\log\ \ (x+1) - \log(x+2)\ ]$$

Differentiating both sides with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\ \left[3.\frac{3}{3x-4} - 4.\frac{1}{x+1} - \frac{1}{x+2}\right]$$

$$\therefore\ \frac{dy}{dx} = \sqrt{\frac{(3x-4)^3}{(x+1)^4\ (x+2)}}\ \text{x}\ \frac{1}{2}\left[\frac{9}{3x-4} - \frac{4}{x+1} - \frac{1}{x+2}\right]$$

(iii) $\int_{-a}^{a} \quad \frac{a-x}{\sqrt{a^2-x^2}}$ dx

$= \int_{-a}^{a} \frac{a-x}{\sqrt{a^2-x^2}}$

$= \int_{-a}^{a} \frac{a-x}{\sqrt{a^2-x^2}}$

SECTION – II

Ans..4 (i) (a) $P(X < 1) = P\ (X = 0) = 0.1$

(b) $P\ (X \geq 3) = P(X = 3) + P\ (X = 4)$

$= 0.15 + 0.25 = 0.4$

(c) $P(1 < X < 4) = P(X=2) + P(X=3)$

$= 0.3+0.15 = 0.45$

(d) $P(2<X<3)\ = P(X = 2) + P\ (X = 3)$

$= \ 0.3 + 0.15 = 0.45$

(ii) $3x - 36 > 0 \Rightarrow\ 3x > 36$

$\Rightarrow\ x\ >\ 12$

$\therefore$ Solution Set is $(12, \infty)$

(iii) Let x kg of zinc be added.

∴ From the given condition we get

$$\frac{\frac{37}{100}(400)+x}{400+x} = \frac{70}{100}$$

$$\therefore \frac{148+x}{400+x} = \frac{7}{10}$$

∴ 10(148 + x) = 7(400 + x)

∴ 1480 + 10x = 2800+7x

∴ 3x = 2800 – 1480

∴ 3x = 1320

∴ x = 440

∴ 440 kg of zinc is added.

(iv) 3 : x = 13 : 104

i.e. $\frac{3}{x} = \frac{13}{104} \Rightarrow x = \frac{3\ x\ 104}{13}$

= 24

(v) Policy value = ₹ 2,00,000

Rate of premium = ₹ 35 per thousand p.a

∴ Amount of premium = $\frac{35}{1,000}$ x 2,00,000

= ₹ 7,000

Rate of commission = 15%

∴ Amount of premium = 7,000 x $\frac{15}{100}$

= ₹ 1,050

(vi) Total no. of deaths = $\sum Di = 900$

$\sum Pi = 9000$ + 25000 + 32000 + 9000 = 75,000

CDR = $\frac{\sum Di}{\sum Pi} \times 1000$

= $\frac{900}{75000} \times 1000 = 12$

(vii) From the given p.m.f. the probability distribution of x is

∴ E (x) $= \sum x \,.\, P(x)$

= - 0.4+0+0.2+0.2 = 0

(viii) The order in which the jobs should be processed

4	1	3	2	5	6

Ans..5 (A) (i) Since amount ₹ 2000 is deposited at the end of every quarter.

∴ It is an immediate annuity

Given C= ₹ 2,000

∴ rate of interest is 8% p.a

∴ rate of interest per quarter = $\frac{8}{4}$ = 2

∴ r = 2% ∴ I = $\frac{r}{100}$ = 0.02

n = no. of quarters = 1 x 4 = 4

∴ n = 4

Using formula of accumulated value A

A = $\frac{C}{i}\,[(1+i)^n - 1]$

= $\frac{2000}{0.02}\,[(1+0.02)^4 - 1]$

= 1,00,000 $[(1.02)^4 - 1]$

= 1,00,000 $[1.024 - 1]$

= 1,00,000 $[0.0824]$

= 8240

∴ Accumulated amount at the end of 1 year is `8240.

(ii) $l_{4\,=\,60,}\ L_4$ = 45. P_4 = ?

$L_x = \frac{l_{x+}\,l_{x+,1,}}{2}$ ∴ $L_4 = \frac{l_{4+}\,l_{5\,,}}{2}$

∴ $45 = \frac{60+\,l_5}{2}$ ∴ 60 + l_5 + 90

∴ l_5 = 30

Again dx = $l_{x+} l_{x+,1}$,

$\therefore$ d_4 = 60 – 30 = 30

We have P_x = 1 - 9_x

$\therefore$ P_4 = 1 - 9_4

= 1 - $\frac{d_4}{l_4}$ $\left(\therefore 9x = \frac{d_x}{l_x}\right)$

= 1 - $\frac{30}{60}$ = 1 – 0.5

$\therefore$ $P_4 = 0.5$

(iii) Step 1 : Note that the number of rows is not equal to number of columns of the above matrix. So the problem us unbalanced. It is balanced by unbalanced. It is balanced by introduction of a dummy job IV with zero cost . This step is done in Table

Subordinates	**Jobs**			
	I	**II**	**III**	**IV**
A	7	3	5	0
B	2	7	4	0
C	6	5	3	0
D	3	4	7	0

Step 2 : "Minimum element of each row is subtracted from every element in that row." Since zero is minimum element for each row, the resultant new matrix is same as given table

Step 3 : " Minimum element in each column is subtracted from every element in that column".

Subordinates	**Jobs**			
	I	**II**	**III**	**IV**
A	5	0	2	0
B	0	4	1	0
C	4	2	0	0
D	1	1	4	0

Step 4 : "Zero elements are covered with minimum number of straight lines"

Subordinates	**Jobs**			
	I	**II**	**III**	**IV**
A	5	0	2	0
B	0	4	1	0
C	4	2	0	0
D	1	1	4	0

Since number of lines covering all zeros is equal to number of rows / columns, the optimal solution has reached.

Optimal assignment can be made as shown in Table

Subordinates	**Jobs**			
	I	**II**	**III**	**IV**
A	5	[0]	2	~~0~~
B	[0]	4	1	~~0~~
C	4	2	4	~~0~~
D	1	1	[0]	[0]

Note that each row, each column contains one assigned zero.So the solution is optimal. Neglecting the assignment of subordinate D to dummy job IV, the following optimal assignment is obtained.

Subordinates	**Jobs**	**Effectiveness**
A	II	3
B	I	2
C	III	3

The total (minimum) effectiveness is = 3 + 2 + 3 = 8 units.

(B) (i) From given data, the LPP is formulated as

Minimize $z = 4x + 6y$

Subject to $x + 2y + \geq 80$.

$3x + y \ \geq 75.$

$x \geq 0\ ,\ y = 0$

Inequation	**Equation**	x	y	**Point**	**Region**
$x + 2y \ \geq 80$	$x + 2y = 80$	0 80	40 0	$(0, 40)$ $(80, 0)$	Non - origin
$3x + y \ \geq 75$	$3x + y = 75$	0 25	75 0	$(0, 75)$ $(25, 0)$	Non - origin

$z = 4x + 6y$

At A (80 ,0) z = 4(80) + 6 (0)

= 320

At C (0, 75) z = 4 (0) + 6 (75)

= 450

At B (14, 33) z = 4 (14) + 6 (33)

z = 56 + 198 = 254

The value of z is minimum at B (14, 33)

∴ 14 Units of chemical A and 33 units of chemical Should be produced.

(ii) Here, we need to obtain line of regression of X on Y which can be expressed as

X = $a' + b_{xy}$ Y

Where b_{xy} = $\frac{cov(X,Y)}{\sigma_y^2}$

= $r\frac{\sigma_x}{\sigma_y}$

= 0.8 $\frac{(3.6)}{(25)}$

= 0.1152

and $a' = \bar{x}$ - b_{xy} $\bar{y}$

= 7.6 – (0.1152) (14.8)

= 5.89504

∴ Line of regression of X on Y is
X = 5.89504 + 0.1152Y

Estimate of X for Y = 10 is

X = 5.89504 + 0.1152Y

X = 7.04704

(iii) Let x Length (cm) and y = weight (gm)

x_i	y_i	xy_i	$x^2{}_i$	$x^2{}_i$
3 4 6 7 10	9 11 14 15 16	27 44 84 105 160	9 16 36 49 100	81 121 196 225 256
30	65	420	210	879

$\sum x_i = 30,\ \sum y_i = 65,\ \sum x_i y_i = 420,$

$\sum x^2 = 210$, $\sum y^2 = 879.$

$\bar{x} \quad \frac{\sum x_i}{n} = \frac{30}{5} = 6 \qquad \bar{y} \quad \frac{\sum x_i}{n} = \frac{60}{5} = 13$

$$r = \frac{\frac{1}{n}\sum x_i y_i \bar{x}\ \bar{y}}{\sqrt{\frac{\sum x^2}{n} - x^{-2}}\sqrt{\frac{\sum y^2}{n} - y^{-2}}}$$

$$= \frac{\frac{1}{5}(420)-(6)(13)}{\sqrt{\frac{210}{5}-(6)^2}\sqrt{\frac{879}{5}-(13)^2}} \qquad = \frac{84-78}{\sqrt{42.36.}\sqrt{175.8-169}}$$

$$= \frac{6}{\sqrt{6}.\sqrt{6.8}} = \frac{6}{\sqrt{40.8}} = \frac{6}{\sqrt{6.3874}} = 0.93934$$

Q.6. **(A)** (i) CDR = $\frac{\sum Di}{\sum pi} \times 1000$

For population A :

$\sum D_i = 170 + 115 + 490 + 630$

= 1405

$\sum P_i$ = 13 + 20 + 52 + 22

= 107 (in thousands)

CDR for population A denoted by CDR_A is

CD$R_A = \frac{\sum Di}{\sum pi} \times 1000$

= $\frac{1405}{107000} \times 1000$

= 13.13 per thousand.

for population B :

$\sum D_i = 510 + 130 + 570 + 680$

= 1890

$\sum D_i$ = 15 + 35 + 54 +23

= 127 (in thousands)

∴ CDR for population B denoted by CDR_B is,

CD$R_B = \frac{\sum Di}{\sum pi} \times 1000$

= $\frac{1890}{127000} \times 1000$

= 14.88 per thousand.

observe that population A is more healthy than population B as

CD$R_A <$ CDR_B .

(ii) Let the equation $2x + 3y - 6 = 0$ be equation of Y on X.

$\therefore 3y = -2x + 6$

$\therefore y = \frac{2}{3}x + 2 \quad \therefore b_{yx} = -\frac{2}{3}$

Another equation $5x + 7y - 12 = 0$ be the eq"of X on Y

$\therefore x = -\frac{7}{5}$ y + $-\frac{12}{5} \qquad \therefore b_{yx} = -\frac{7}{5}$

Correlation coefficient r = $\pm\sqrt{b_{yx}.\ b_{xy}}$

$= \pm\sqrt{\left(-\frac{2}{3}\right)\ \left(-\frac{7}{5}\right)}$

$= \pm\sqrt{\frac{14}{15}} \qquad = \sqrt{0.9333} \qquad$ = - 0.96607

$\therefore$ r = 0.96607

To Calculate $\frac{\sigma_x}{\sigma_y}$ we know that

$b_{xy} = r\frac{\sigma_x}{\sigma_y} \quad \therefore -\frac{7}{5} = 0.96607\ \frac{\sigma_x}{\sigma_y}$

$\frac{\sigma_x}{\sigma_y} = \frac{7}{3(0.96607)} = \frac{7}{2.89821}$ = 2.4152

(iii) Here min M_1 = 3, min M_3 = 5 and max M_2 = 5.

Since min $M_3 \geq$ max M_2 is satisfied, the problem can be converted into 7 job 2 machines problem. Thus if G and H are the two fictitious machines such that

G = M_1 + M_2

and H = M_2 + M_3

then the problem can be written as the following 7 job and 2 machines problem.

Job	**A**	**B**	**C**	**D**	**E**	**F**	**G**
Machine G	7	11	9	9	10	12	10
Machine H	10	10	7	16	6	10	15

Using the optimal sequence algorithm, the following optimal sequence can be obtained.

A	D	G	F	B	C	E

For total elapsed time, we have

Total elapsed time T = 59 hrs

Idle time for machines

M_1 = 59 – 46 = 13 hrs

M_2 = 59 – 22 = 37 hrs.

M_3 = 59 – 52 = 07 hrs.

(B) (i) Let the fixed monthly salary = ₹ x Rate of commission = r %

$\therefore$ Income = salary + 64000 $\times \frac{r}{100}$

Also Income = salary + 72000 $\times \frac{r}{100}$

$\therefore$ 10650 = $x + 640r$ ....(1)

11450 = $x + 7250r$ ....(2)

By taking (2) – (1)

$\therefore$ 800 = $80r$

$\therefore r$ = 10

Putting this value in eqn (1) we get

$\therefore 10650 = x + 640\ (10)$

$\therefore 10650 = x + 6400$

$\therefore\ \ x = 10650 - 6400 = 4250$ ₹

(ii) Let us give ranks to values of X and Y assigning rank 1 to the highest values and next highest be ranked 2 etc.

X	**Y**	**Rank of X** x_i	**Rank of Y** y_i	$d_i = x_{i=} y_i$	$d_i^{\ 2}$
50	110	8	9.5	-1.5	2.25
55	110	6.5	9.5	-3.0	9.00
65	115	3.5	7	-3.5	12.25
50	125	8	4	4.0	16
55	140	6.5	2	4.5	20.25
60	115	5	7	-2.0	4.00
50	130	8	3	5.0	25
65	120	3.5	5	-1.5	2.25
70	115	2	7	-5.0	25
75	160	1	1	0.0	0.00
				Total	116.00

Here, in series X, 50 is repeated thrice, 55 is repeated twice. 65 is repeated twice, In series. In series Y 110 is repeated twice and 115 is repeated thrice.

$\therefore T_x = \frac{3\ (3^2-1)}{12} + \frac{2\ (2^2-1)}{12} + \frac{2\ (2^2-1)}{12}$ = 3

$T_y = \frac{2\ (2^2-1)}{12} + \frac{3\ (3^2-1)}{12} = 2.5$

Corrected $\sum\ d_i^{\ 2} = \sum\ d_i^{\ 2}\ + T_x\ + T_y$

= 116 + 3 + 2.5

= 121.5

$\therefore$ R = 1 - $\frac{6[corected \sum \; d_i^2]}{n(n^2-1)}$

= 1 - $\frac{6\times 121.5}{10(10^2-1)}$

= 1 - $\frac{729}{990}$ = $\frac{268}{990}$ = 0.26

(iii) Let x $=$ No. of members successfully treated.

$p =$ Probability that treatment is effective.

$\therefore p = 70\% = 0.7$

$q = 1 - p = 1 - 0.7 = 0.3 \,.n = 4$

(i) Exactly two members are successfully treated.

P [X = 2] = $n_{C_x}\, p^x\, q^n - x$

$\therefore$ P [X = x] = 4_{C_2} (0.7) ^{2} (0.3) $^{4-2}$

= $\frac{4\times 3}{2\times 1}\times$ 0.49 $\times$ 0.09

= 6 $\times 0.49 \;\times 0.09 = 0.49 \;\times 0.54 = 0.2646$

(ii) At least one member is successfully treated.

$\therefore$ P [X ≥ 1] = 1 – P [X = 0]

= 1 - 4_{C_0} (0.7) ^{0} (0.3) ^{4}

= 1 – 1 $\times$ 1 $\times 0.0081$

= 1 – 0.0081 = 0.9919

(iii) All are successfully treated.

P [X = 4] = 4_{C_4} (0.7) ^{4} (0.3) $^{4-4}$

= 1 $\times 0.2401 \times 1$

= 0.2401